ŠOLA LOGIKE

Dve logični uganki

V tem sestavku bomo rešili dve logični uganki iz knjige R. Smullyana *The lady or the tiger* (A. Knopf, New York, 1982). Slovenski prevod je izšel leta 1989 pri DZS.

Problem 1. Janez in Peter sta brata dvojčka. Janez je zagrešil kaznivo dejanje. Vemo še, da vsaj eden od dvojčkov vedno laže. Sodnik vpraša enega od bratov, ali je Janez. Ta pritrdi in sodnik vpraša enako še drugega. Ta odgovori "*DA*" ali "NE" in sodnik ve, kateri je Janez. Kateri je torej Janez?

Problem 2. Neki logik je obiskal otok resničnikov in neresničnikov. Srečal je dve osebi: A in B. Vprašal je osebo A, ali sta oba resničnika. A je odgovoril na vprašanje, toda to še ni bilo dovolj, da bi logik ugotovil, kaj sta otočana. Nato je še enkrat vprašal osebo A – tokrat, ali sta oba istega tipa (to je – oba resničnika ali oba neresničnika). Spet je A odgovoril ("*DA*" ali "*NE*") in logik je vedel, kaj sta. Kaj sta A in B?

Rešitev prvega problema:

Vpeljimo naslednje oznake:
$P \equiv$ *Prvi vprašani vedno laže.*
$D \equiv$ *Drugi vprašani vedno laže.*
$J \equiv$ *Prvi je Janez.*

Potem $\neg J$ pomeni, da je drugi Janez. Torej se je drugo vprašanje glasilo:"*Ali velja* $\neg J$*?*". Dejstvo, da vsaj eden od bratov vedno laže, zapišemo simbolično $P \lor D$.Če prvi vedno laže, potem je njegov odgovor, da je on Janez, napačen:

$$P \Rightarrow \neg J$$

Za drugega imamo dve možnosti:

a) Da je odgovoril "*DA*". Potem imamo še $D \Rightarrow J$.

b) Da je odgovoril "*NE*". Tedaj imamo še $D \Rightarrow \neg J$.

Toda mi ne vemo, ali se je zgodilo *a)* ali *b)*. Vemo pa, da je sodnik na osnovi podatkov lahko izpeljal, kateri je Janez.

V primeru *a)* semantično drevo izgleda takole:

$$\begin{array}{c} P\lor D \\ P\Rightarrow\neg J \\ D\Rightarrow J \\ \begin{array}{c|c} P & D \\ \neg J & J \end{array} \end{array}$$

Torej je možno tako J kot $\neg J$. Na osnovi teh podatkov sodnik ne bi mogel izpeljati, kdo je Janez.

V primeru *b)* drevo izgleda takole:

$$\begin{array}{c} P\lor D \\ P\Rightarrow\neg J \\ D\Rightarrow \neg J \\ \begin{array}{c|c} P & D \\ \neg J & \neg J \end{array} \end{array}$$

Na obeh vejah nastopa $\neg J$. To je, iz $\{P\lor D, P\Rightarrow\neg J, D\Rightarrow\neg J\}$ logično sledi $\neg J$. Simbolično to zapišemo:

$$P\lor D, P\Rightarrow\neg J, D\Rightarrow\neg J \models \neg J$$

Prvi torej ni Janez.

Rešitev drugega problema:

Vpeljimo naslednje oznake:
$A \equiv$ Oseba A je resničnik.
$B \equiv$ Oseba B je resničnik.
$A\land B \equiv$ Oba sta resničnika.
$A\Leftrightarrow B \equiv$ Oba sta istega tipa.

Recimo, da je odgovor na prvo vprašanje "*NE*". Potem imamo

$$\begin{array}{c} A\Leftrightarrow\neg(A\land B) \\ \begin{array}{c|c} \begin{array}{c} A \\ \neg(A\land B) \\ \begin{array}{c|c} \neg A & \neg B \\ \times & \\ & \end{array} \end{array} & \begin{array}{c} \neg A \\ A\land B \\ A \\ B \\ \times \end{array} \end{array} \end{array}$$

V tem primeru bi logik vedel, kaj sta, torej je odgovor "*DA*". Recimo, da je odgovor na drugo vprašanje tudi "*DA*":

$$\begin{array}{c} A\Leftrightarrow A\land B \\ A\Leftrightarrow(A\Leftrightarrow B) \\ \begin{array}{c|c} \begin{array}{c} A \\ A\land B \\ B \\ \\ \\ \\ \\ \end{array} & \begin{array}{c} \neg A \\ \neg(A\land B) \\ \begin{array}{c|c} \begin{array}{c} \neg A \\ \neg(A\Leftrightarrow B) \\ \begin{array}{c|c} A & \neg A \\ \neg B & B \\ \times & \end{array} \end{array} & \begin{array}{c} \neg B \\ \neg(A\Leftrightarrow B) \\ \begin{array}{c|c} \neg A & A \\ B & \neg B \\ \times & \times \end{array} \end{array} \end{array} \end{array} \end{array} \end{array}$$

Še vedno imamo dve možnosti $\{A, B\}$ oziroma $\{\neg A, B\}$. Odgovor na drugo vprašanje je "*NE*".

$$
\begin{array}{c}
A \Leftrightarrow A \wedge B \\
A \Leftrightarrow \neg(A \Leftrightarrow B)
\end{array}
$$

$$
\begin{array}{c|c}
\begin{array}{c}
A \\
A \wedge B \\
B \\
\neg(A \Leftrightarrow B) \\
\begin{array}{c|c}
A & \neg A \\
\neg B & B \\
\times & \times
\end{array}
\end{array}
&
\begin{array}{c}
\neg A \\
\neg(A \wedge B) \\
\begin{array}{c|c}
\neg A & \neg B \\
A \Leftrightarrow B & A \Leftrightarrow B \\
\neg B & \neg A
\end{array}
\end{array}
\end{array}
$$

Imamo eno samo možnost za resničnost podatkov: $\{\neg A, \neg B\}$.

URNIK TEKMOVANJ V LETU 1994

področje	**šola**	**tekmovanje**	**datum**
matematika	osnovna	šolsko	do 9. aprila
		občinsko	20. april
		državno	21. maj
	srednja	izbirno	19. marec
		državno	23. in 24. april
		olimpiada	julij
fizika	osnovna	šolsko	do 26. marca
		področno	9. april
		državno	7. maj
	srednja	izbirno	16. april
		državno	14. maj
		olimpiada	junij
logika	osnovna in srednja	izbirno	september
		državno	oktober
matematika za razvedrilo	osnovna in srednja	državno	3. september
računalništvo	osnovna	šolsko	25. marec
		področno	22. ali 23. april
		državno	21. maj
	srednja	državno	13. - 14. maj
raziskovanje inovatorstvo	srednja	državno	27. maj

LOGIKA PO SVETU: The Lie Detective

With a twinge od apprehension such as he had never felt before, an anthropologist named Abercrombie stepped onto the Island of Knights and Knaves. He knew that this island was populated by most perplexing people: knights, who make only true statements, and knaves, who make only false ones. "*How*," Abercrombie wondered, "*am I ever to learn anything about this island if I can't tell who is lying and who is telling the truth?*"

Abercrombie knew that before he could find out anything he would have to make one friend, someone whom he could always trust to tell him the truth. So when he came upon the first group of natives, three people, presumably named Arthur, Bernard, and Charles, Abercrombie thought to himself, "*This is my chance to find a knight for myself.*" Abercrombie first asked Arthur, "*Are Bernard and Charles both knights?*" Arthur replied, "*Yes.*" Abercrombie then asked: "*Is Bernard a knight?*" To his great surprise, Arthur answered: "*No.*" Is Charles a knight or a knave?

Abercrombie knew that he must first determine what type (knight or knave) Arthur and Bernard are. Arthur is obviously a knave, since no knight would claim that Bernard and Charles are both knights yet deny that Bernard is a knight. Therefore both of Arthur's answers were lies. Since he denied that Bernard is a knight, Bernard really is a knight. Since he affirmed that both Bernard and Charles are knights, it is false that they are both knights; at least one of them must be a knave. But Bernard is not a knave (as we have proved), therefore, Charles must be a knave.

Abercrombie was then informed by one of the three he knew to be a knight that the island had a sorcerer.

"*Oh, good!*" Abercrombie exclaimed. "*We anthropologists are particularly interested in sorcerers, witch doctors, medicine men, shamans and the like. Where do I find him?*"

"*You must ask the King,*" came the reply.

Well, the anthropologist was able to obtain an audience with the King and told him that he wished to meet the Sorcerer.

"*Oh, you can't do that,*" said the King, "*unless you first meet his apprentice. If the Sorcerer's Apprentice approves of you, then he will allow you to meet his master; if he doesn't, then he won't.*"

"*The Sorcerer has an apprentice?*" asked the anthropologist.

"*He certainly does!*" replied the King. "*There is a famous musical composition about him – I believe the composer was Dukas. Anyway, if you wish to meet the Sorcerer's Apprentice, he is now at his home, which is the third house on Palm Grove. At the moment he is entertaining two guests. If, when you arrive, you can deduce which of the three present is the Sorcerer's Apprentice, I believe that will impress him sufficiently that he will allow you to meet the Sorcerer. Good luck!*"

A short walk brought the anthropologist to the house. When he entered, there were indeed three people present.

"*Which of you is the Sorcerer's Apprentice?*" asked Abercrombie.